UNIVERSITY OF MIAMI
ROSENSTIEL SCHOOL of MARINE, ATMOSPHERIC & EARTH SCIENCE

Estimating Oceanic Kinetic Energy: Comparing Eulerian and Lagrangian Approaches

Eli Faigle[1], Shane Elipot[1]

[1]Rosenstiel School of Marine, Atmospheric, and Earth Science, University of Miami, FL

1) Research Objectives and Motivation

Why is it important map/quantify the kinetic energy of ocean currents?

1) They play an important role in both ocean conditions and atmospheric climate (Elipot & Wenegrat, 2021).
2) Their maps are used for:
   1) Understanding global ocean dynamics
   2) Assessing the accuracy of ocean models
   3) Understanding dispersion of particles and pollutants

Objectives:

Follow-up Arbic et al. (2022), who found discrepancies between in situ (Lagrangian) drifter data and (Eulerian) numerical simulations by:

1) Calculating oceanic kinetic energy (KE) through Eulerian and Lagrangian approaches from a high-resolution model simulation
2) Conducting analyses, comparisons, and attempting to quantify Lagrangian bias
3) Identifying phenomena which deserve following up

2) Data

The data originate from the HYbrid Coordinate Ocean Model (HYCOM), a global model with 1/25 degree resolution, 41 vertical hybrid layers, forced by 3-hour atmospheric fields for one year (2014). This state-of-the-art model simulates both the general ocean circulation and ocean tides.

Datasets analyzed:

A. Eulerian hourly velocities at 0 m and 15 m from the HYCOM non-regular 7055x9000 grid (~11 TB), divided into 60-day long time series, every 30 days from January 1.

B. Lagrangian hourly velocities from numerical particles (synthetic drifters) released offline within HYCOM:

- Released on ¼ degree grid at 0 m and 15 m depths using the *Ocean Parcels* software
- Advected for 60 days, from 11 starting dates every 30 days starting from January 1, totaling ~ 11M trajectories (~ 356 GB)

Tools:

Parallel computations were conducted on UM Triton Supercomputer using Python scripts and open-source software (clouddrift.org). All data are publicly available on cloud storage at *arn:aws:s3:::hycom-global-drifters*.

3) Methods

Spectral Analysis:

To estimate spectra as seen in **Fig. 2**, the periodogram estimator was used for each 60-day long time series and then averaged accordingly.

Velocity Filtering:

Lowpass and bandpass filtering were achieved with a Morse wavelet transform applied to complex velocity time series. This amounts to extracting the energy for each 'ridge' in the zonally-averaged spectra of **Fig. 2:** Semidiurnal (SD) (±[1.85, 2.15] cpd), diurnal (D) (±[0.85, 1.15] cpd), inertial (-[0.85, 1.15]f cpd), (f = Coriolis frequency) and low-frequency (LF) (<0.5 cpd).

Kinetic Energy (KE) estimation:

- Velocity variance, interpreted as KE, is calculated in 1/2 degree lon/lat bins from either Lagrangian or Eulerian velocity estimates in each bin.

- Lagrangian and Eulerian KE maps are compared by calculating and mapping the normalized ratio statistic:

$$\frac{Lagrangian\ KE - Eulerian\ KE}{Lagrangian\ KE + Eulerian\ KE}$$

This statistic is 0 when the two KE are equal; is > 0 (<0) if Lagrangian KE is larger (smaller) than Eulerian KE.

4) Results

1) Zonally-Averaged Spectra

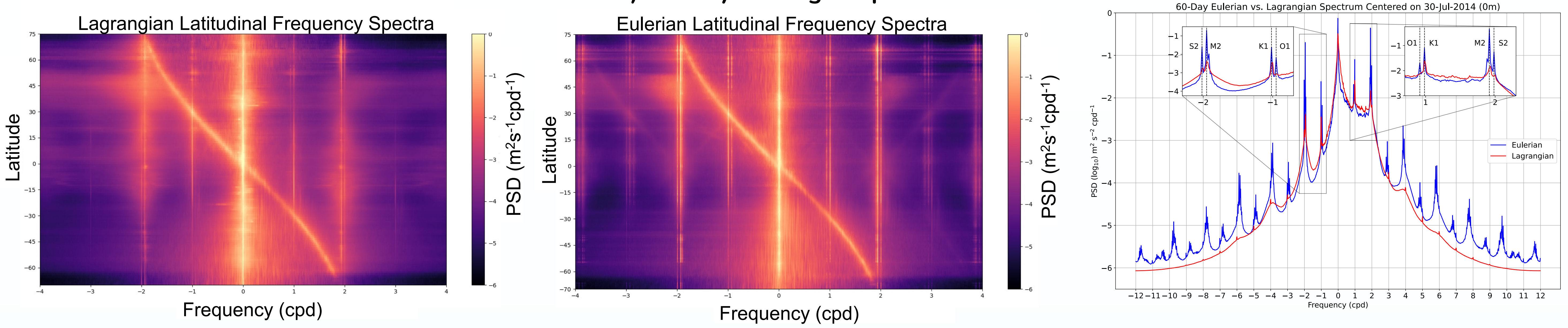

Fig. 2: Lagrangian (left) and Eulerian (middle) zonally-averaged rotary spectra in ½ degree latitudinal bins. Lagrangian tidal energetic peaks appear 'smeared', and the Lagrangian spectrum loses peaks at higher frequencies. Globally averaged rotary velocity spectra (right), where anticyclonic and cyclonic frequencies are assigned to positive and negative frequencies, respectively. All figures are centered on July 30, 2014, at 0 meters.

2) Eddy Rings

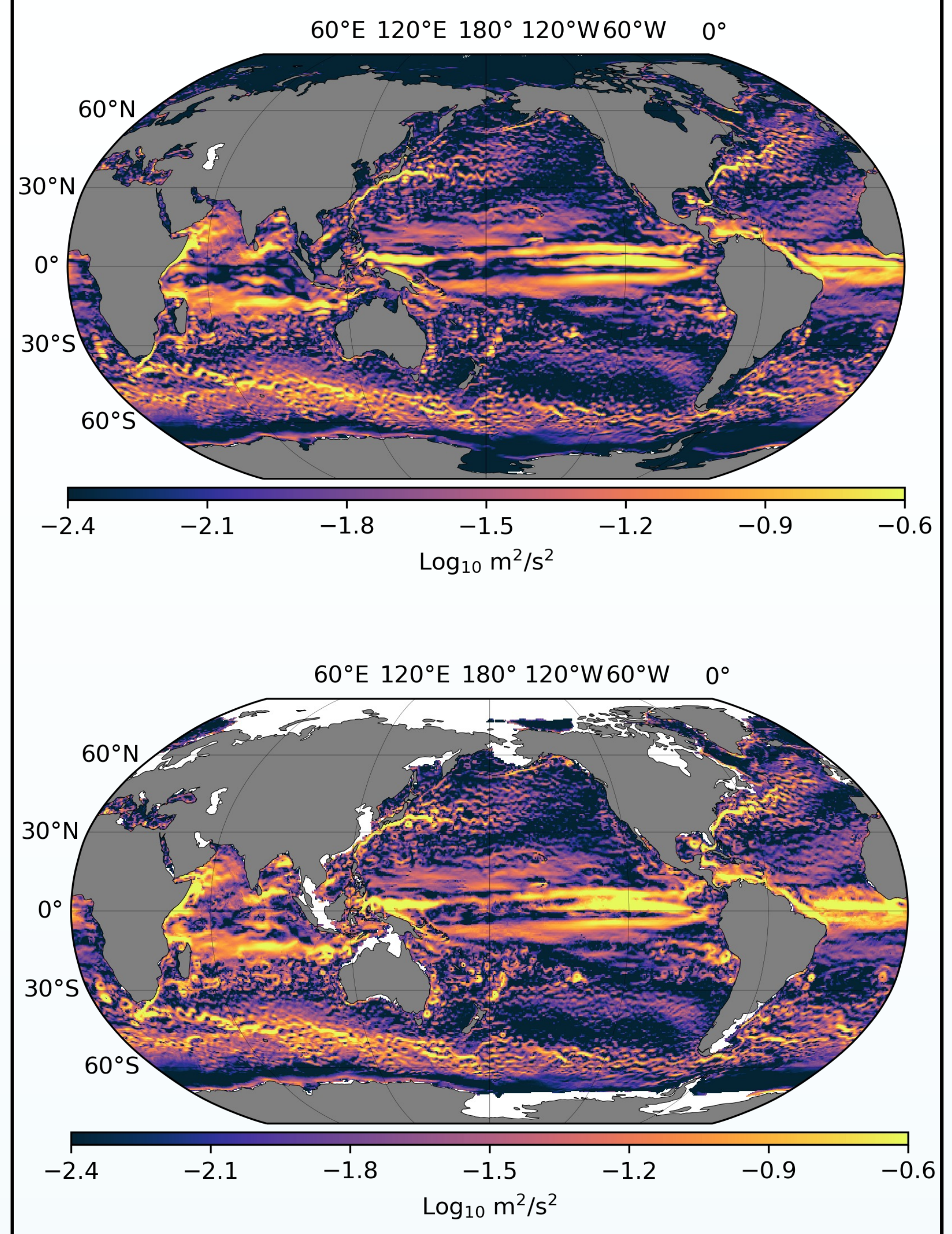

Fig. 3: Lagrangian LF mean kinetic energy (MKE) (top) and Eulerian LF MKE (bottom). The eddy rings are 'smeared' in the Eulerian map. Both are centered on July 30, 2014, at 0 meters.

3) Semidiurnal Band

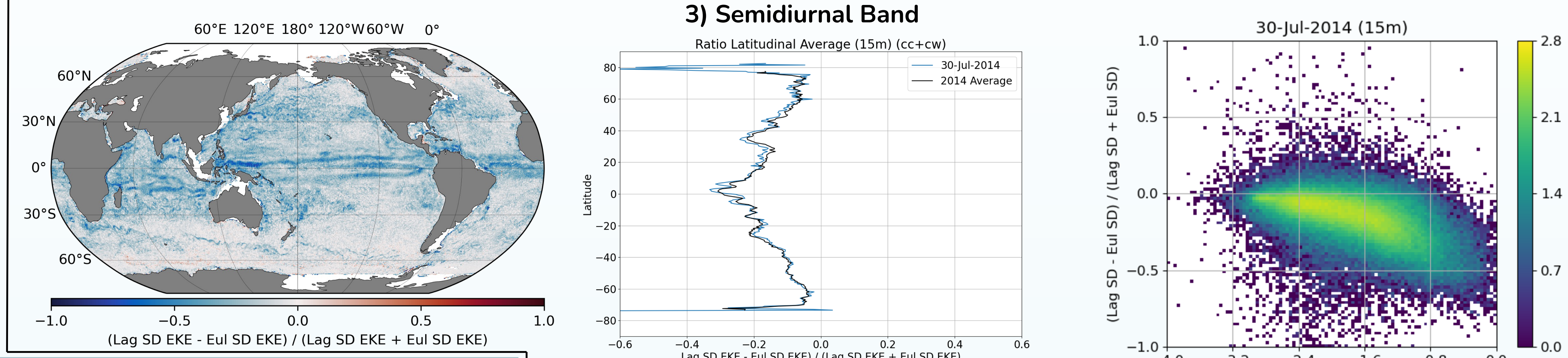

Fig. 4: Global map of the ratio statistic for SD EKE (left), the ratio statistic averaged by latitude (middle), and 2-D histogram (right) illustrating the relationship between LF TKE and the SD ratio. All centered on July 30, 2014, at 15 meters depth.

4) Inertial Band

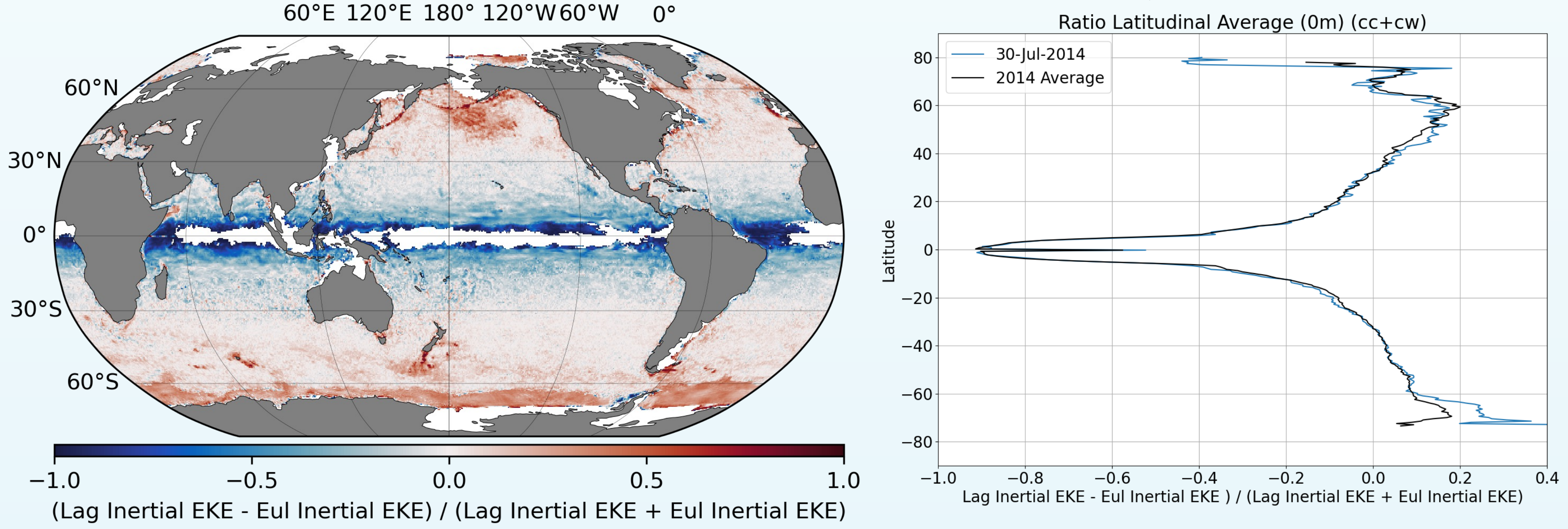

Fig. 5: Global map of the ratio statistic for the inertial band (left) and the ratio statistic averaged by latitude (right), each centered on July 30, 2014, at 0 meters. The ratio appears to be a function of latitude.

5) Conclusions

		Lagrangian = x * Eulerian	
		0 meters	15 meters
Tidal Frequency	LF TKE	1.1075	1.1739
	LF EKE	1.1209	1.2124
	LF MKE	1.064	1.0367
	SD	0.6849	0.7182
	D	0.9724	0.9512
	Inertial	0.8975	0.9029
	Unfiltered	0.994	1.0121

Table: Conversion coefficients from globally-averaged ratio statistic values from all 11 datasets at 0- and 15-meter depths. Only results within 60S-60N are considered to avoid sea ice in the model.

Takeaways:

- The relationship between Lagrangian and Eulerian KE is a function of frequency.
- The identified biases indicate that it is advisable to exercise caution when estimating SD KE with drifters.
- The relationship between the Lagrangian biases at 0 and 15 meters is dependent on the frequency band.

Acknowledgements

I would like to thank Shane Elipot for bringing me onto his project, guiding me through the research process, and providing me with the experience needed to complete this project. I'd also like to thank Michael Brown and Josefina Olascoaga for being a part of this process and providing me insightful feedback.

Shane Elipot and Eli Faigle were supported by NSF award number 1851166. Shane Elipot was additionally supported by a Provost Research Award from the University of Miami.

References

Arbic et. al, https://doi. org/10.1029/2022JC018551

Elipot, S., & Wenegrat, https://doi.org/10.5065/ybca-0s03

Hogan, 10.5670/oceanog.2014.73.